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# The Utility Standard and the Patentability of Intermediate Technology

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# The Utility Standard and the Patentability of Intermediate Technology

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**Abstract**

We explore the consequences of the utility requirement on speed of innovation and welfare. A weak utility requirement means that an intermediate technology with no immediate application or commercial value is patentable. Using a model of two stage innovation with free entry and trade secrecy, we identify cases when patentability is beneficial to society. Although a firm may undertake basic research protected by trade secrecy, patentability is still desirable when spillover is high and innovation costs are high. However, patentability becomes less desirable as basic research costs decrease. We also show that high value of final technology by itself does not favor non-patentability and identify condition when it does.

**JEL Classification:** O34 O31

**Key Words:** utility requirement, basic research, patentability, innovation

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# 1 Introduction

Utility, together with novelty and the inventive step (or non-obviousness), constitutes the three basic requirements for patentability. It requires that the invention can bring about a specific technical effect. When research is directly guided by "real-world" necessities, it is easy to establish the utility of inventions. However, when it is driven by scientific discovery, it may be an "intermediate technology", the real world utility of which can be determined only after further research. For instance, the immediate application of a gene sequence or a new chemical entity may not be clear without substantial further research. The utility requirement may reject patentability of such an intermediate technology. A weak utility requirement implies that the intermediate technology is patentable.

Despite the increasing importance of the utility standard in science-driven innovations, there are almost no substantive economic analysis of the standard. Harhoff, Regibeau and Rockett (2001) is one exception in this regard, although there have been some legal analysis of this issue, notably Grady and Alexander (1992), Merges (1997), and Heller and Eisenberg (1998). The purpose of this paper is to present a framework and analyze the welfare implications of the utility standard.

The economic rationale of the utility standard can be best clarified in the context of cumulative innovation. Although similar in structure, our research issue and our formulation differ from previous cumulative innovation analysis in several ways. First, past studies on novelty standard and forward protection in the context of cumulative innovation have focused on the patentability of the follow-up invention and the infringement possibility of such an invention on the prior invention (Scotchmer and Green (1995), Denicolo (2000), to name a few). That is, the first stage invention is assumed to be patentable and has a stand alone value. In our analysis, the first stage innovation is an intermediate technology and further research is necessary to realize its potential value. The issue for us is to analyze whether the high standard of utility standard which can reject the patentability of such intermediate technology is welfare increasing or not, while assuming patentability of the second stage invention. By definition of intermediate technology, the second stage invention always infringes on the first stage invention, when the latter is patented.

Secondly we incorporate both trade secrecy and spillover. With intermediate technology, involuntary disclosure is unlikely because the technology is used only for further research. It may remain within the confines of a building or limited number of people within the inventing firm. Thus trade secret protection is available even if patent protection is not. However trade secrecy loses its protective power once competitors obtain the technology independently or if there is unintentional spillover. And spillovers often occur through academic publications and contacts among researchers, both of which are significant in science-driven innovations.

Since we assume free entry in the first stage basic research, the rent obtained from the commercialization of the final technology is dissipated, irrespective of the patentability of intermediate research. In the patentability case, it will be dissipated more in the first stage (larger first stage expenditures and more entries in first stage competition). We show that preventing dissipation of the rent in the second stage through a patent is welfare improving even if trade secrecy enables the first stage basic research, particularly if spillover is very likely, or basic research is very expensive. In addition, the patentability improves welfare when intermediate technologies require very high investment in the development stage for commercialization, so that entry in the first stage is barely profitable without patentability.

The relative advantage of patentability declines as basic research costs become smaller. When the marginal cost of basic research becomes very small, we show that patentability reduces welfare. This is because when the cost of basic research is low, it is more welfare-improving to promote development by eliminating market power. Thus relaxing the utility requirement for intermediate technologies that are mere "ideas" would not be socially desirable.

We also show that high value of the final technology by itself does not favor strong utility requirement i.e., rejection of the patentability of the intermediate technology, even if appropriation with trade secrecy protection is effective. Patentability of the intermediate technology improves welfare even under such circumstance if the marginal research cost of the first stage is sufficiently large compared to the marginal cost of the second stage or if the interest rate is sufficiently high. This is because both high interest rate and low second stage marginal cost cause high investment in the second stage even under monopoly. Thus welfare loss from patentability and the resulting market power is limited under these circumstances.

In the remainder of this section, we present a brief background and issues regarding the utility requirement and review the previous works we feel our work is most closely related. We also clarify the difference between the utility and the novelty standards. The main analysis based on a two-stage patent race is in Section 2 followed by a section on welfare. We discuss the implications of the major assumptions and extensions in Section 4. We conclude with Section 5 with policy implications of our results.

### Utility and description requirements

Section 101 of the U.S. Patent Law stipulates the utility requirement by the following statement "Whoever invents any new and useful process, machine, manufacture, improvement thereof, or composition of matter … may obtain a patent therefore ...”[1] The recent guidelines of the USPTO interprets that Section 101 requires that “an invention must be supported by a specific, substantial and credible utility ...” According to the guideline, utility specific to the subject matter, instead of general utility, has to be claimed. Utilities that require or constitute carrying out further research to identify or reasonably confirm a “real world” context of use are not substantial utilities. In addition, an assertion is credible unless the logic underlying the assertion is seriously flawed, or the facts upon which the assertion is based are inconsistent with the logic underlying the assertion. The utility requirement is also implicit in Section 112, which requires written descriptions of the invention and of the manner and process of making and using it without undue experimentation.”

Traditionally, utility requirement has been an issue in the chemical industry. In this industry, research may yield synthesized compounds for which no particular use is known. A 1966 U.S. Supreme Court ruling (“Brenner ruling”) supports the denial of the patent for such compounds if it fails to disclose any utility, even though it is closely related to another compound which is useful. [2] However this ruling is considered to represent the “high-water mark” of utility doctrine (Merges (1997)). The recent ruling in re Brana in 1995 seems to be based on logic conflicting the previous Supreme Court Ruling. It established that utility for pharmaceutical products can be established by animal testing.[3]

[1]The law of nature, physical phenomena, and abstract ideas are not regarded to be patentable subject matter.

[2]“Until the process claim has been reduced to production of a particular product shown to be useful, the metes and bounds of that monopoly are not capable of precise delineation. It may engross a vast, unknown, and perhaps unknowable area. Such a patent may confer power to block off whole areas of scientific development, without compensating benefit to the public. The basic quid pro quo contemplated by the Constitution and the Congress for granting a patent monopoly is the benefit derived by the public from an invention with substantial utility. ⋯ But a patent is not a hunting license. It is not a reward for the search, but compensation for its successful conclusion.” Brenner v. Manson, 383 U. S. 519, 148 U.S.P.Q. (BNA) 689 (1966)

[3]“FDA approval, however, is not a prerequisite for finding a compound useful within the meaning of patent laws. Usefulness in patent law, and in particular in the context of pharmaceutical inventions, necessarily includes the expectation of further research and development. The stage at which an invention in this field becomes useful is well before it is ready to be

More recently, utility and enablement requirement has become a big issue in biotechnology industry where innovation are driven by scientific progress. Recent scientific advances have resulted in intermediate technology such as identified gene sequences. This is critical for but only useful by making further research possible. In applying for a patent on partial genetic sequences (expressed sequences tags or EST) in 1991, the NIH (Dr. Craig Venter) claimed that these can be used as diagnostic probes, identification of chromosomes, etc, which are uncertain general utilities. The NIH gave up patenting in 1994, when it faced a rejection by the USPTO based on utility and other requirements, as well as strong criticism from scientific and the other circles. (See Aoki and Nagaoka (2002) for more on biotechnology and the utility standard.)

The patentability of research results is especially critical for the firms specialized in research, very important players in the U.S. biotechnology industry. Since these firms do not have internal assets to implement downstream research such as clinical testing, patents for intermediate research results are essential for them to sell the research outputs or to attract investment money for engaging in downstream research. The head of the leading U.S. biotechnology venture firm states that "Some argue that the invention is not complete until the precise biological activity of an individual gene is identified; indeed, there is some indication that the Patent Office intends to apply the new guidelines in this way. This argument ignores the real world utility, described above, associated with the isolation, sequencing and identification of genes and their classification into categories whose general functions are known. If this standard were to apply, then only those companies that adhered to the inefficient, vertically-integrated pharmaceutical industry model would be entitled to patents. This approach

administered to humans. Were we to require Phase II testing in order to prove utility, the associated costs would prevent many companies from obtaining patent protection on promising new inventions, thereby eliminating the incentive to pursue, through research and development, potential cures in many crucial areas such as the treatment of cancer." In re Brana 51 F. 3d 1560, 34 U. S. P. Q. 2d 1436 (Fed. Cir. 1995) would be at odds with the evolution of the pharmaceutical industry, with its attendant efficiencies."(Testimony of Randal Scott, president and chief scientific officer of Incyte Genomics Inc., before the U.S. House Judiciary Subcommittee on Courts and Intellectual Property, July 13, 2000) We discuss the application of our analysis to such an "outside" innovator in Section 5.

The utility standard can also become an issue with concept patents. That is, the patenting of a general product or business ideas that use new technology. The concept is novel but is a mere idea acquired at very little cost. It has little role in advancing knowledge, but which has to be used widely in applying a new technology. Such a concept patent would discourage R&D investment for using the new technology, since it enables the patentee to collect royalty, but does not aid R&D at all in terms of knowledge. Such an invention may be rejected based on a non-obviousness requirement, but can also be rejected based on the absence of specific utility.

**Existing literature**

Matutes, Regibeau, and Rocket (1996) also explore optimal patent policy in a two stage innovation process where the first stage is basic research and has no stand alone value. They explore the trade-off between disclosure and protection of basic research. Our paper focuses on trade-off between first and second stage innovations (Chang (1995), O'Donoghue, Scotchmer and Thisse (1998), O'Donoghue (1999), Denicolo, V. and P. Zanchettin (2002) and others. See Scotchmer (forthcoming) for overview of sequential innovation). However because of free-entry in each stage, strength (including no protection) not just shift the profit between first and second stage but how much rent is dissipated at each stage.

In this section we review four papers that we believe our work is most closely related. Grossman and Shapiro (1987) analyze whether firms support patentability of intermediate technology in the framework of a two-stage race among duopolists, in which the completion of the first stage research is necessary for commencing the second stage research but the first stage research has no commercial value. Based on simulations, they suggest that intermediate patent may be beneficial to the firms ex-post (i.e., after the first stage research), but not ex-ante, since it intensifies competition. They assume that an intermediate patent requires the competing firm to drop out of the second stage research race, so that the second stage research is always a monopoly. They do not consider the possibility of trade secret protection and their focus is strictly firm's incentive and overall welfare is not analyzed.

Scotchmer and Green (1990) analyze the novelty standard with respect to the interim innovation also in a duopoly framework. The focus of their analysis is the role of the patent in facilitating disclosure, which accelerates research in their model. They take into account the possibility that a firm chooses trade secret protection for intermediate technology even if it is patentable. They find that a weak novelty requirement promotes disclosure while it does not undermine ex-ante profit significantly, and that the first-to-file regime encourages disclosure more than the first-to-invent regime (see Aoki and Nagaoka (2002) Appendix for how these findings can be carried over to the case of the utility standard). Their analysis, however, cannot be considered as an analysis of the utility standard in the context of cumulative innovation, for the following two reasons. First, they assume that the second innovation does not infringe the first innovation, even if the latter is patented. However, in those cases where the utility standard is an issue with respect to the patentability of the intermediate technology, the second innovation infringes the first innovation, if the latter is patented, since the first innovation provides a crucial input to the second stage research. Second, they assume that the intermediate technology can have a direct commercial value and can compete with the final innovation. Obviously this is not the case when utility of the first stage technology is in question.

Denicolo (2000) analyzes the optimal degree of forward protection of the first innovation in the framework of a two-stage patent race with free entry in both stages. He analyzes the economic effects of the patentability of the secondary innovation and its potential infringement of the first innovation, or the degree of forward protection, assuming the patentability of the first stage innovation. He shows that strong forward protection becomes less attractive as the relative profitability of the first innovation increases and the relative difficulty of obtaining it decreases. Although we use and extend his analytical framework, we address a very different issue. We analyze the economic consequences of the patentability of the first innovation by comparing the case where the first innovation is patentable under the weak standard of utility and the case where it is not patentable due to the strong standard of utility so that it can only be protected by trade secret. In terms of structure of the model, although the first case (the patentable case) becomes equivalent either to UI (the secondary innovation is unpatentable and infringing) or PI (the secondary innovation is patentable and infringing) in the Denicolo analysis, the second case where the first innovation is protected only by trade secret is completely out of the scope of his analysis. In addition, we incorporate fixed cost of research in the analysis, since duplicative aspects or economy of scale may be important especially in the development stage of innovation.

Finally, Harhoff, Regibeau and Rockett (2001) analyze the effect of the patentability of the first stage innovation in the framework of a two-stage R&D race in the context of genetically modified food. They show that gene patents (the patentability of the first stage innovation) causes inefficient stockpiling of gene patents when interest rate is low. This result, however, depends on very restrictive supply response of the firms as assumed in their analysis: fixed R&D resources of the duopoly incumbent firms and no entry.

## 2 The Model

We assume free entry into both the first basic research stage (R stage) and the second development stage (D stage) innovation competition unless it is constrained by patent protection or trade secrecy. Unlike Denicolo, we assume that it is possible for a firm to resort to trade secrecy to protect the intermediate technology. This is a viable option because the technology is used only for the purpose of further research.

However the shortcoming of trade secret protection for a firm is that it does not prevent rivals from using the same technology if it is obtained independently. This is one of the essential differences between trade secrecy and patent protection. Thus a firm using trade secret protection faces potential competition in the second stage. (In fact with Poisson discovery process, another firm will succeed the R stage with probability one.) Since we assume that research expenditure in each stage is completely sunk once commenced, there is no reason for a firm in R stage to drop out of competition when another firm has completed the R stage, unless it believes that it cannot profitably enter the D stage research competition.

We assume that an intermediate technology is either a type that spills over completely or a type that does not. We denote by $\gamma$ the probability that the technology is the type that spills over. This probability is common knowledge. Once the R stage is completed, i.e., a firm obtains the intermediate technology successfully, the firm knows immediately which type the technology is. If the technology is the spillover type, spillover occurs immediately unless it is protected by a patent.[4] In this case D stage will be competitive with free entry. If the technology is the no spillover type (which is the case with probability $1-\gamma$), then trade secrecy will be effective unless technology is obtained independently.

Specifically, firm $i$ chooses research intensity $x_{it}$ for cost $c_t$ for R&D at stage $t$, where $t = R$ or $t = D$. Discovery in each stage follows a Poisson process. We assume there is a fixed cost $f_t$ to participate in stage $t$. If the intermediate technology is patentable, then the patentee will be the sole developer of the final technology.[5] Because it is an intermediate technology, there is no direct commercial value to the result of the R stage innovation.[6] The value of the final technology is $v$.

We consider two cases, when the intermediate technology is patentable and when it is not. If it is patentable, whoever succeeds the R stage has a choice of patenting. The regime when the intermediate technology is not patentable is the same as the no patenting decision even when the technology is patentable.

## 2.1 D Stage investment

We will first analyze the D stage investment behavior under the two regimes. We characterize the equilibrium investments, the patenting choice and the corresponding profits.[7]

**The intermediate technology is Patentable**

We first characterize the equilibrium investment when the firm has a patent on the intermediate technology ($P$). There is no spillover because patent protec-

[4] Successful completion is observed by all firms and thus other firms will also know immediately which type the technology is.

[5] Because of the Poisson discovery process, there is no advantage to licensing and having many firms engage in R&D. Of course a firm may be forced to license if it does not possess resources to engage in D stage. This case is discussed in Section three. Even in this case, the particular innovation technology implies there should only be one licensee

[6] This is equivalent to Denicolo's $UI$ or $PI$ with $v_1 = 0$.

[7] D stage constitutes a subgame of the two stage game. The equilibrium we characterize is part of a subgame perfect Nash equilibrium strategy.

tion is perfect. It will be shown later that a firm always prefers to patent the intermediate technology if this is legally possible.

When the firm has the patented technology, it is able to invest as a monopolist. It chooses $x$ to maximize,

$$\int_0^\infty exp^{-(x+r)\tau} xv d\tau - c_D x - f_D = \frac{xv}{x+r} - c_D x - f_D.$$

Since the second order condition is satisfied, the monopoly investment, $x_m$, is as follows:

$$x_m = \sqrt{\frac{rv}{c_D}} - r,$$

and the monopoly profit is,

$$\pi_m = \left(\sqrt{v} - \sqrt{c_D r}\right)^2 - f_D. \tag{1}$$

We assume that this is always positive,

$$(\sqrt{v} - \sqrt{c_D r})^2 > f_D. \tag{2}$$

The equilibrium D stage profit when the intermediate technology is patented is, $\pi_D^P = \pi_m$ and the corresponding investment is $X_D^P = x_m$.

**The intermediate technology is Not Patentable**

When the intermediate technology is not patentable ($N$), there are two subgames after completion of the R stage, depending on the type of technology: one with spillover (probability $\gamma$) and one without (probability $1 - \gamma$). If there is spillover, the firm must compete with new entrants in the D stage on equal footing. If there is no spillover, the firm can invest to exploit the first mover advantage.

We start with the case with spillover. There are $n$ firms (the number determined in equilibrium) in D stage competition. We follow the methodolgy of Denicolo (1999). Firm $i$'s profit when its investment is $x_i$ is,

$$\pi_i = \int_0^\infty exp^{-(\sum_{j=1}^n x_j + r)\tau} x_i v d\tau - c_D x_i - f_D = \frac{x_i v}{\sum_{j=1}^n x_j + r} - c_D x_i - f_D. \quad (3)$$

Since the second order condition is satisfied, the following first order condition for profit maximization characterizes the profit maximizing investment:[8]

$$\frac{\partial \pi_i}{\partial x_i} = v \frac{\sum_{j \neq i} x_j + r}{(x_i + \sum_{j \neq i} x_j + r)^2} - c_D = 0. \quad (4)$$

There will be an incentive to invest a positive amount when this marginal profit is positive at $x_i = 0$ which is the case by virtue of assumption (2).

In symmetric equilibrium with free entry, profit given by (3) should equal 0 and $x_j = x$ for all $j$. Equations (3) and (4) become

$$\frac{xv}{nx + r} - c_D x - f_D = 0,$$
$$\frac{(n-1)x + r}{(nx + r)^2} v - c_D = 0.$$

The two equations characterize the equilibrium investment and the equilibrium number of firm.

The equilibrium investment is

$$x_0 = \frac{\sqrt{f_D v} - f_D}{c_D}.$$

Ignoring the integer problem, we have the equilibrium number of firms engaged

[8] All summation hereafter will be for $i = 1, \ldots, n$ unless noted $j \neq i$ which is for $j = 1, \ldots, i-1, i+1, \ldots n$.

in D stage investment,

$$n_0 = \sqrt{\frac{v}{f_D}} - \frac{c_D r}{\sqrt{f_D v} - f_D}.$$

Number of firms is decreasing in both fixed and marginal costs. Investment by each firm is also decreasing in marginal cost but will be increasing in fixed cost if fixed cost is sufficiently small relative to value of technology,

$$\frac{dx_0}{df_D} = \frac{\sqrt{v} - 2\sqrt{f_D}}{c_D} \gtreqless 0 \quad \sqrt{v} \gtreqless 2\sqrt{f_D}. \tag{5}$$

Larger fixed cost can increase or decrease investment of each firm, depending on which effect is larger: less firms or more investment per firm. The total investment with spillover is however always decreasing in both costs,

$$X_0 = n_0 x_0 = \frac{v - \sqrt{v f_D}}{c_D} - r.$$

The equilibrium profit when there is spillover is zero, i.e., $\pi_S = 0$.

If there is no spillover, the firm acts as an incumbent in D stage anticipating entry. It invests to such an extent that even an entrant expecting no further entries cannot make money. Although we focus on the entry deterrence strategy in the following analysis, the major conclusions of the analysis would apply in the case of the entry accommodation strategy as discussed in Section 4. The firm chooses $x$ to deter entry. An entrant's profit when it invests $x_e$ is,

$$\pi_e = \int_0^\infty exp^{-(x_e+x+r)\tau} x v d\tau - c_D x - f_D = \frac{x_e v}{x_e + x + r} - c_D x_e - f_D. \tag{6}$$

The entrant will invest to maximize this profit, given incumbent's investment

$x$. That is, $x_e$ satisfies the first order condition,

$$\frac{\partial \pi_e}{\partial x_e} = v\frac{x+r}{(x_e+x+r)^2} - c_D = 0.$$

The incumbent will choose $x$ so that profit $\pi_e$ will be zero even when the entrant is profit maximizing. The entry deterrent output, $x_b$ is,

$$x_b = \frac{(\sqrt{v}-\sqrt{f_D})^2}{c_D} - r.$$

$x_b > x_m$ for

$$\frac{(\sqrt{v}-\sqrt{f_D})^2}{\sqrt{v}} > \sqrt{rc_D}. \tag{7}$$

This condition requires that the fixed cost not be too large and is also a sufficient condition for $\pi_m \geq 0$. If this condition does not hold, then entry will be blocked with monopoly investment. Note that $x_b \to X_0$ as $f_D \to 0$: entry deterrence is impossible if there is no fixed cost.

The equilibrium profit with entry deterrence will be,

$$\begin{aligned}\pi_b &= v - (\sqrt{v}-\sqrt{f_D})^2 - c_D r\left(\frac{v}{(\sqrt{v}-\sqrt{f_D})^2} - 1\right) - f_D \\ &= 2\sqrt{f_D}(\sqrt{v}-\sqrt{f_D}) - c_D r\left(\frac{v}{(\sqrt{v}-\sqrt{f_D})^2} - 1\right).\end{aligned} \tag{8}$$

The entry deterrence profit is decreasing in $c_D$. It is also decreasing in the fixed cost, $f_D$, when it is large but increasing in fixed cost when $f_D$ is small, relative to $v$. While larger $f_D$ means it is possible to deter entry with smaller deviation from the monopoly profit, it also directly reduces profit (including the monopoly profit). The positive effect dominates only when $f_D$ is small.

Summarizing, investment ($x_{NS}$) and profit ($\pi_{NS}$) when there is no spillover are $x_b$ and $\pi_b$ if (7) holds, and $x_m$ and $\pi_m$ if (7) does not hold. Hereafter we assume that condition (7) holds. The equilibrium D stage profit of the firm successful in R stage[9] as anticipated at the beginning of the game (taking into account that Nature determines type of technology) when the intermediate technology is not patentable is,

$$\pi_D^N = \gamma\pi_S + (1-\gamma)\pi_{NS} = \gamma 0 + (1-\gamma)\pi_b. \tag{9}$$

$\pi_D^N$ is always less than $\pi_m$ for any probability of spillover $\gamma$ and strictly less for $\gamma > 0$. We make the following observation about relative size,

**Lemma 1.** *Assuming that condition (7) holds, then*

$$x_m < x_b = x_{NS} < X_0, \quad \pi_m > \pi_b = \pi_{NS} > \pi^N > \pi_D^N > 0.$$

Both $x_b$ and $X_0$ are linear in $v$ (maximum order is $v$) but $x_m$ is order of $\sqrt{v}$. Reduction of research investment due to monopoly power increases with value of the final patent, $v$. Since $\pi_D^P = \pi_m > \pi_D^N$, due to (7) and the positive probability of spillover ($\gamma > 0$),

**Corollary 1.** *A firm will always patent the intermediate technology if it is patentable.*

In our framework patent enforcement is perfect and there is no spillover related to patenting. Even if trade secret protection is perfect, it offers no protection against independent innovation. This alone makes patent protection always more attractive.

[9]The other firms' profits are zero.

## 2.2 R stage investment

**General solution of R stage**

We derive a general solution for R stage when the payoff to the winner from the D stage is $p_D$ and losers get nothing. Firm $i$'s expected payoff when it invests $x_i$ and other firms invest $x_j$ is,

$$\pi_i = \frac{x_i \pi_D}{x_i + \sum_{j \neq i} x_j + r} - c_R x_i - f_R. \tag{10}$$

First order condition for profit maximization is,

$$\frac{\partial \pi_i}{\partial x_i} = \frac{\sum_{j \neq i} x_j + r}{(x_i + \sum_{j \neq i} x_j + r)^2} \pi_D - c_R = 0. \tag{11}$$

There will be an incentive to invest a positive amount when this marginal profit is positive at $x_i = 0$ which will hold if $\pi_D > c_R r$.

Again, using symmetry,[10] we obtain the equilibrium investment:

$$x_R = \frac{\sqrt{f_R \pi_D} - f_R}{c_R}.$$

In order for this to be positive (interior solution), profit from the next stage must be sufficiently large, $\pi_D > f_R$. Investment is decreasing in marginal cost and increasing in D stage profit $\pi_D$. The effect of fixed cost on investment is analogous to (5). Ignoring the integer problem, we have the equilibrium number

[10] In symmetric equilibrium with free entry, (10) should equal 0 and $x_j = x$ for all $j$. Equations (10) and (11) become

$$\frac{x \pi_D}{nx + r} - c_R x - f_R = 0,$$
$$\frac{(n-1)x + r}{(nx + r)^2} \pi_D - c_R = 0.$$

The two equations characterize the equilibrium investment and number of firms.

of firms engaged in R stage investment,

$$n_R = \sqrt{\frac{\pi_D}{f_R}} - \frac{c_R r}{\sqrt{f_R \pi_D} - f_R}.$$

The number of firms is also decreasing in both costs and increasing in D stage profit $\pi_D$. The aggregate investment, $X_R$ is,

$$X_R(\pi_D) = n_R x_R = \frac{\sqrt{\pi_D}}{c_R}\left(\sqrt{\pi_D} - \sqrt{f_R}\right) - r, \tag{12}$$

if $\pi_D$ is sufficiently large and $X_R = 0$ otherwise. This is also increasing in D stage profit. The equilibrium investments when the intermediate technology is patentable, $X_R^P$, and when not patentable, $X_R^N$ can be found by substituting the appropriate equilibrium profits from D stage, $\pi_D^P$ and $\pi_D^N$. Equation (12), together with Lemma 1 shows how investment is increased in one stage at the cost of reducing it in the other.

**Proposition 1.** *Patentability of the intermediate technology increases R stage research investment but reduces D stage investment.*

From (12), we can make the following observation:

**Lemma 2.** *When the intermediate technology is not patentable, spillover must be sufficiently unlikely and costs ($c_D$,$c_R$, $f_R$) small enough for there to be investment in the intermediate technology. That is,*

$$X_R^N > 0 \quad \Leftrightarrow \quad \sqrt{(1-\gamma)\pi_b} > \frac{\sqrt{f_R}}{2} + \sqrt{c_R r + \frac{f_R}{4}}. \tag{13}$$

Recall that D stage equilibrium profit $\pi_D$ will be low when D stage marginal cost is high. This lemma shows that if D stage marginal cost is increased, there must be a corresponding decrease in R stage costs, or there will no investment in basic research . Not surprisingly, very high likelihood of spillover results in no R stage investment.

# 3 Welfare

The value of technology $v$ is the firm's private value. This does not capture the additional value to society from the innovation which we denote by $s$. Given aggregate investment $X$,

$$P(X) = \frac{X}{X+r},$$

is the "adjusted probability" of innovating (Denicolo (2000)). It discounts the value according to the delay which is distributed according to a Poisson process. The expected welfare is, denoting investments in R stage and D stage by $X_R$ and $X_D$,

$$W(X_R, X_D) = P(X_R)\left\{P(X_D)(v+s) - c_D X_D - n_D f_D\right\} - c_R X_R - n_R f_R.$$

From Lemma 2, we can immediately identify a case when patentability will unambiguously improve welfare.

**Corollary 2.** *If there is no R stage investment without patentability and if there is with patentability, then patentability will improve welfare.*

There will be no R stage investment when condition (13) does not hold in which case welfare will be zero. The condition is violated when costs in either stage is very large. Given that a firm can recover investment in R stage research only from commercialization of D stage innovation, not only high cost of R stage research but also high marginal cost of D stage research tends to favor patentability of the intermediate technology. Thus if an intermediate technology requires a large amount of additional work (high investment costs) for commercialization, this would be precisely the situation when making the intermediate technology patentable will improve welfare.

Noting that profit is bid down to zero in equilibrium through both stages of competition, the welfare with and without patentability of the intermediate technology are,

$$W^P = P(X_R^P)P(X_D^P)s = P(X_R(\pi_m))P(x_m)s,$$
$$W^N = P(X_R^N)\left\{\gamma P(X_0) + (1-\gamma)P(x_b)\right\}s = P(X_R((1-\gamma)\pi_b))\left\{\gamma P(X_0) + (1-\gamma)P(x_b)\right\}s.$$

Superscripts $N$ and $P$ denote when intermediate technology is "not patentable" and "patentable".

An iso-welfare curve in $(X_R, X_D)$ space is depicted in Figure 1 for $\gamma = 0$ and $X_D = x_b$. Convexity can be derived as in Denicolo (2000). The figure demonstrates the trade-off involved in making intermediate technology patentable. Patentability increases $X_R$ and reduces $X_D$ (Proposition 1). In the figure, this means patentability will change investments in the direction of the arrows.

We begin with establishing the following relationship, analogous to Proposition 5 of Denicolo (2000).

**Lemma 3.** *The ratio $W^P/W^N$ is (i) increasing in $c_R$ and (ii) increasing in $f_R$.*

Proof is in the Appendix.

This ratio ($W^P/W^N$) being greater or less than 1 determines if welfare is higher or lower with patentability. The exact conditions are established in the following propositions. First, we characterize the relationship between R&D costs and the welfare effect of patentability using Lemma 3.

**Proposition 2.** *(i) Patentability of intermediate technology improves social welfare if marginal cost of basic research is very high and reduces it if marginal* *cost of basic research is very low. More generally, there is a $c_R^*$ such that*

$$W^P \gtreqqless W^N \Leftrightarrow c_R \gtreqqless c_R^*. \tag{14}$$

*(ii) Patentability of intermediate technology improves welfare if fixed cost of basic research is very high, so that development research with patentability becomes barely profitable. That is, there is a $f_R^*$ such that*

$$f_R > f_R^* \Rightarrow W^P > W^N.$$

*(iii) Similarly, patentanbility always improves social welfare when marginal and fixed costs of development are large. That is,*

$$W^P > W^N \; for \; \begin{cases} sufficently\ large\ c_D, \\ sufficienlty\ large\ f_D. \end{cases}$$

Proof is in the Appendix.

The expression (12) implies that $c_R r$ being close to $\sqrt{\pi_D^N}(\sqrt{\pi_D^N} - \sqrt{f_R})$ means R stage investment $X_R$ is very small. In Figure 1 it would be a point such as $T$, a point at which the change in investments from patentability improves welfare. On the other hand, small $c_R$ implies $X_R$ is large, such as point $S$ in Figure 1. Social welfare depends on the product of the adjusted probability of D stage success and that of R stage success. As a result, when the probability R stage success is high due to lower research cost of that stage (low $c_R$), it is more efficient to encourage the expansion of the D stage reward. Since patentability reduces the D stage adjusted probability, non-patentability becomes more advantageous.

Monotonicity of $\frac{P(X_R^P)}{P(X_R^N)}$ with respect to $f_R$ and $c_R$ implies that the critical value $c_R^*$ is decreasing in $f_R$. The range of R stage marginal cost for which patentability is undesirable becomes smaller when the fixed cost is larger.

We now characterize the relationship between extent of possible spillover and the welfare effect of patentability. Using (9) and Lemma 1, the adjusted probability for R stage is, for any $\gamma$,

$$P(X_R^N) = P(X_R((1-\gamma)\pi_b)) < P(X_R(\pi_D^P)) = P(X_R(\pi_m)).$$

$P(X_R^N)$ is decreasing in $\gamma$ and approaches zero as $\gamma$ approaches unity. On the other hand,

$$\gamma P(X_0) + (1-\gamma)P(x_b) > P(X_D^P) = P(x_m),$$

holds for any $\gamma$. Greater spillover benefits society at the D stage but it has an adverse effect on R stage investment. Using (1), (8), and (12), we are able to identify the minimum $\gamma$ above which patentability of the intermediate technology is beneficial to society.

**Proposition 3.** *Patentability of intermediate technology always improves social welfare when spillover is very large. That is, there is always a level $\gamma^P$ such that for all $\gamma \geq \gamma^P$ the following holds,*

$$W^P > W^N.$$

Proof is in the Appendix.

Although spillover increases D stage investment, profit is dissipated by free entry. This will reduce the incentive to invest in the R stage. Note that this is independent of the size of fixed costs.

We synthesize the previous propositions by the following proposition pertaining to large $v$.

**Proposition 4.** *When the final technology is very valuable, patentability is* *desirable if and only if the following condition holds:*

$$\sqrt{rc_R} > 2(1-\gamma)\sqrt{c_D f_D}.$$

*That is, only if the above condition holds,*

$$W^P > W^N,$$

*for sufficiently large $v$.*

Proof is in the Appendix.

The proposition shows that the high value of the final technology by itself does not determine if the patentability of intermediate technology is desirable or not, despite appropriation via trade secrecy. We can interpret the above inequality in the following manner. When the value of the final technology is high, whether the patentability is desirable or not depends only on the ratio between $X_D^P$ and $X_R^N$, which are the levels of investments of the respective critical stages of the patentability and non-patentability regimes. When interest rate $r$ is high or $c_D$ is low, the investment in the development stage is high, even when the intermediate technology is patentable and the second stage innovation is monopolized. The monopoly investment increases as $r$ increases because high interest rate induces a monopoly firm to realize the invention quickly so as to avoid heavy discounting. Therefore the patentability is desirable. On the other hand, when $f_D$ is high so that the development stage profit is high under non-patentability regime,[11] or when $c_R$ is low, the investment in the first stage is high even when the intermediate technology is not patentable. This makes the patentability of intermediate research undesirable. In sum, balancing the incentives for the two stages still matter even if the final technology has a very

[11]Equation (8) shows that the profit of the firm successful in the research stage increases with $f_D$ when $v$ is very large (equation (5)).

high value. Conditions on $\gamma$ and $c_R$ are consistent with Propositions 2 and 3.

# 4 Discussions of the implications of major assumptions and extensions

We have developed the analysis, assuming that intermediate technology owner is an integrated firm, able to engage in D stage innovation. If only independent innovators can engage in R stage research and only the ex-post licensing is feasible, patentability of intermediate technology becomes socially more desirable since such firm must share the profit from the D stage research with the licensee under most circumstances. If the patentee appropriates all the rent, our analysis follows, including the welfare results. This would be the case if there is free entry into the licensee market, or if the patentee is able to make a take or leave it offer. Any other license bargaining (strategic or Nash Bargaining) will result in the independent inventor's rent being reduced which weakens R stage incentive.

While we assumed that the firm successful in the research stage adopts the entry deterrence strategy in the non-patentability case (i.e. when the intermediate technology is protected only by trade secret), our main conclusions should hold when the firm adopts the entry accommodation strategy from the following observation. It is natural for us to assume that the aggregate investment in the D stage ($X_D$) without patentability is higher than with patentability (i.e. $x_m < X_a < X_0$), even if the firm adopts an entry accommodation strategy, as in the case when it adopts entry deterrence strategy. Moreover, D stage competition due to non-patentability reduces both the expected revenue from entry and the difference in profits of the winner and the losers of the R stage competition relative to the case with patentability, so that the aggregate investment in the

R stage would be lower without patentability. This provides a reason why the claim of Proposition 1 would continue to hold. Corollary 2 also holds since the welfare can become positive only if $P(X_R)$ is positive.

As for Proposition 2, equation (14) relies on the monotonicity of the ratio $W^P/W^N$ (Lemma 3) which depends on the functional form. However the proposition will still be true for extreme values of costs. As the marginal or fixed costs of R stage research becomes large, the investment in that stage declines toward zero, since entry in the research itself will become unprofitable eventually. Such threshold costs continue to be lower with non-patentability, since competition in the D stage reduces the expected profit from the entry. On the other hand, the reduction of investment due to the monopolization of the D stage investment continues to be bounded. Thus patentability of basic research becomes welfare improving. On the other hand, as the marginal cost of R stage research ($c_R$) tends to zero, investment level in that stage would increase indefinitely even if the winning firm adopts the entry accommodation strategy. This is because the additional investment by a firm always increases its profit (i.e the marginal revenue remains positive) by increasing the chance of early discovery. Thus, the ratio of the adjusted probabilities for the R stage is close to 1 for a very small $c_R$. Consequently, patentability of intermediate research becomes welfare reducing for such $c_R$. Finally it is self evident that Proposition 3 also continues to hold.

As for the last proposition, high fixed cost of research $f_D$ will no longer favor non-patentability when the firm adopts the entry accommodation strategy. The investment in the R stage with non-patentability increases as the development stage profit increases. That profit, however, can decline as $f_D$ increases when the firm adopts the entry accommodation strategy (this would be the case for the ranges of $f_D$ where the number of firms which enter in the development competition is independent of $f_D$). Thus, while high interest rate, high $c_R$ and low $c_D$ would continue to make patentability desirable as in the case of entry deterrence strategy, high $f_D$ can favor patentability.

## 5 Concluding Remarks

We can derive several policy implications from our analysis. The implications of Proposition 2 is that even if trade secrecy protects intermediate technology, patentability is still beneficial if research costs are high. Such technoology's possibility of spillover reinforces the case for patentability (Proposition 3). On the other hand, patentability should be rejected when the intermediate technology covers a mere "idea" that is easy to acquire. Given the high value of final technology, Proposition 4 suggests, that reduction of the marginal cost of basic research relative to the marginal cost of development, due to, for example, subsidy or tax breaks, makes unpatentability of intermediate technology more desirable. We also showed that high interest rate is more likely to make the patentability of intermediate technology desirable (Proposition 4). It follows that when the intermediate technology results in a very valuable product, society with high interest rate benefits from weak utility requirement.

Because of constant returns to scale nature of innovation of our model, having more firms engage in innovation will not increase the return from innovation. This means a patentee firm capable of doing D stage innovation itself (a vertically integrated firm) will not gain by licensing to another firm to do D stage innovation. If the patentee is unable to do D stage innovation itself (an independent inventor or a vertically unintegrated firm), it will not gain by licensing to more than one firm. If there is to be multiple licensing, it would have to be compulsory licensing (see Aoki and Nagaoka (2002) for how it works). Such compulsory licensing can introduce D stage competition while not totally destroying D stage profit. Thus, it may provide an efficient balance between non-patentability and patentability under certain circumstances.

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# Appendix

### Proof of Lemma 3

*Proof.* In the following, $X(\theta)$ means $X$ is a function of parameter $\theta$ which is either $c_R$ or $f_R$. Then,

$$\frac{dP(X_R)}{d\theta} = \frac{dX_R}{d\theta}\frac{r}{(X_R+r)^2}.$$

Given that $\frac{dP(X_D)}{d\theta} = 0$, we have the following:

$$\frac{d\ln(W^P/W^N)}{d\theta} = \frac{dP(X_R^P)/d\theta}{X_R^P} - \frac{dP(X_R^N)/d\theta}{X_R^N}$$

Using (12),

$$\frac{dX_R}{dc_R} = -\frac{X_R+r}{c_R}.$$

Thus, we have

$$\frac{dP(X_R)}{dc_R} = -\frac{r}{c_R(X_R+r)}.$$

Since $X_R^N < X_R^P$, we have $-dP(X_R^N)/dc_R > -dP(X_R^P)/dc_R > 0$. It follows that

$$\frac{d\ln(W^P/W^N)}{dc_R} > 0.$$

Similarly,

$$\frac{dX_R}{d\sqrt{f_R}} = -\frac{X_R+r}{\sqrt{\pi_D}-\sqrt{f_R}},$$

so that

$$\frac{dP(X_R)}{d\sqrt{f_R}} = -\frac{r}{(\sqrt{\pi_D}-\sqrt{f_R})(X_R+r)}.$$

Since $X_R^N < X_R^P$ and $\pi_D^N < \pi_D^P$, we have

$$-dP(X_R^N)/d\sqrt{f_R} > -dP(X_R^P)/d\sqrt{f_R} > 0.$$

□

**Proof of Proposition 2**

*Proof.* We first show that the reduction of welfare due to decline in D stage investment caused by the monopolization of D stage research is bounded from below. Let us define $k$ as satisfying $v = rc_D(1+k)^2$, which provides a measure of profitability of the final patent relative to the marginal cost of development. From characterizations of $X_0$, $x_b$ and $x_m$, we have,

$$X_0, x_b \le \frac{v}{c_D} - r = r(1+k)^2 - r = (k^2+2k)r, \quad x_m = r(1+k) - r = rk.$$

Together we have,

$$\frac{X_D^P}{X_D^N} \ge \frac{rk}{(k^2+2k)r} = \frac{1}{k+2}. \tag{15}$$

From Lemma 1 (condition (7) holds), we have

$$X_0, x_b > x_m,$$

which implies, [12]

$$\frac{X_D^N + r}{X_D^P + r} > 1.$$

Together with (15) we have,

$$\frac{P(X_D^P)}{P(X_D^N)} > \frac{1}{(k+2)}. \tag{16}$$

[12] Recall D stage investment with no patenting (or not patent) was $X_0$ with spillover and $x_b$ without. $X_D^N$ is defined by

$$P(X_D^N) = \gamma P(X_0) + (1-\gamma)P(x_b).$$

From monotonicity of the function $P(\cdot)$, $x_b < X_D^N < X_0$.

Using (12) we have

$$\frac{P(X_R^P)}{P(X_R^N)} = \frac{X_R^P}{X_R^N} \times \frac{r + X_R^N}{r + X_R^P}$$
$$= \frac{\sqrt{\pi_D^P}(\sqrt{\pi_D^P} - \sqrt{f_R}) - c_R r}{\sqrt{\pi_D^N}(\sqrt{\pi_D^N} - \sqrt{f_R}) - c_R r} \times \frac{\sqrt{\pi_D^N}(\sqrt{\pi_D^N} - \sqrt{f_R})}{\sqrt{\pi_D^P}(\sqrt{\pi_D^P} - \sqrt{f_R})}. \tag{17}$$

The expression is 1 when $c_R r = 0$, increasing in $c_R r$ in the range $c_R r < \sqrt{\pi_D^N}(\sqrt{\pi_D^N} - \sqrt{f_R})$, and approaches infinity as $c_R r \to \sqrt{\pi_D^N}(\sqrt{\pi_D^N} - \sqrt{f_R})$. Note that $\pi_D^N = (1-\gamma)\pi_b$ is independent of $c_R$. For sufficiently large $c_R$,

$$\frac{P(X_R^P)}{P(X_R^N)} > (k+2).$$

Then using (16), we have for such $c_R$,

$$\frac{W^P}{W^N} = \frac{P(X_R^P)}{P(X_R^N)} \frac{P(X_D^P)}{P(X_D^N)} > 1.$$

To show existence of $c_R^*$, we need to show that for sufficiently small $c_R$, the ratio becomes less than 1. From Proposition 1, we have $X_D^P < X_D^N$, and thus $\frac{P(X_D^P)}{P(X_D^N)} < 1$. From (17), we have $\frac{P(X_R^P)}{P(X_R^N)} > 1$ converging to 1 as $c_R$ approaches zero. Monotonicity of $W^P/W^N$ (Lemma 3) implies existence of $c^*$. This ends proof of part (i). A similar argument when $\sqrt{f_R}$ approaches $\sqrt{\pi_D^N} - \frac{c_R r}{\sqrt{\pi_D^N}}$ shows existence of $f_R^*$ which proves part (ii). Part (iii) follows from similar argument of showing that $\sqrt{\pi_D^N}(\sqrt{\pi_D^N} - \sqrt{f_R}) - c_R r$ in (17) goes to zero when $c_D$ or $f_D$ becomes sufficiently large and close to upper bound given by (9). Only caveat is that $k$ depends on $c_D$ meaning when $c_D$ becomes large, the lower bound of (16) changes. Fortunately it moves to make the constraint less binding (right hand side declines). Thus we can still use the bounds and we get the desired inequality. Since we are not able to claim monotonicity of $W^P/W^N$ with respect to development stage costs, we do not have a critical value as in parts (i) and (ii). □

**Proof of Proposition 3**

*Proof.* Since $P(X_R^N)$ is decreasing in $\gamma$ and approaches 0, there is always a $\gamma^P > 0$ such that

$$P(X_R^P)P(X_D^P) = P(X_R((1-\gamma)\pi_b))P(X_0).$$

For any $\gamma \geq \gamma^P$,

$$P(X_R((1-\gamma)\pi_b))P(X_0) > P(X_R((1-\gamma)\pi_b))\left\{\gamma P(X_0) + (1-\gamma)P(x_b)\right\}.$$

. □

**Proof of Proposition 4**

*Proof.* The following approximation holds for large $X$,[13]

$$P(X) = \frac{X}{X+r} \approx 1 - \frac{r}{X}. \tag{18}$$

For small $\theta_1$ and $\theta_2$, we have the following approximation,

$$\frac{1-\theta_1}{1-\theta_2} \approx 1 - \theta_1 + \theta_2. \tag{19}$$

Using (18) and (19), we have for sufficiently large $X_R^N$, $X_D^N$, $X_R^P$, and $X_D^P$,

$$\frac{W^P}{W^N} = \frac{P(X_R^P)}{P(X_R^N)} \times \frac{P(X_D^P)}{P(X_D^N)} \approx 1 + r\left(\frac{1}{X_R^N} - \frac{1}{X_R^P} + \frac{1}{X_D^N} - \frac{1}{X_D^P}\right).$$

[13] Approximations are derived by ignoring all terms of order greater than $\frac{1}{X^2}$. The approximation can be arbitrarily close to the original expression by choosing $X$ sufficiently large.

Although all ivestment levels are increasing in $v$, convergence speeds of the reciprocals differ. We can make the following approximations for large $v$,

$$\frac{1}{X_R^N} \approx \frac{c_R}{2(1-\gamma)\sqrt{f_D v}}, \quad \frac{1}{X_R^P} \approx \frac{c_R}{v}, \quad \frac{1}{X_D^N} \approx \frac{c_D}{v}, \quad \frac{1}{X_D^P} \approx \frac{1}{\sqrt{\frac{rv}{c_D}}}.$$

Thus for sufficiently large $v$,

$$\frac{W^P}{W^N} \approx 1 + \frac{r}{\sqrt{v}} \left( \frac{c_R}{2(1-\gamma)\sqrt{f_D}} - \sqrt{\frac{c_D}{r}} \right) > 1.$$

$\square$

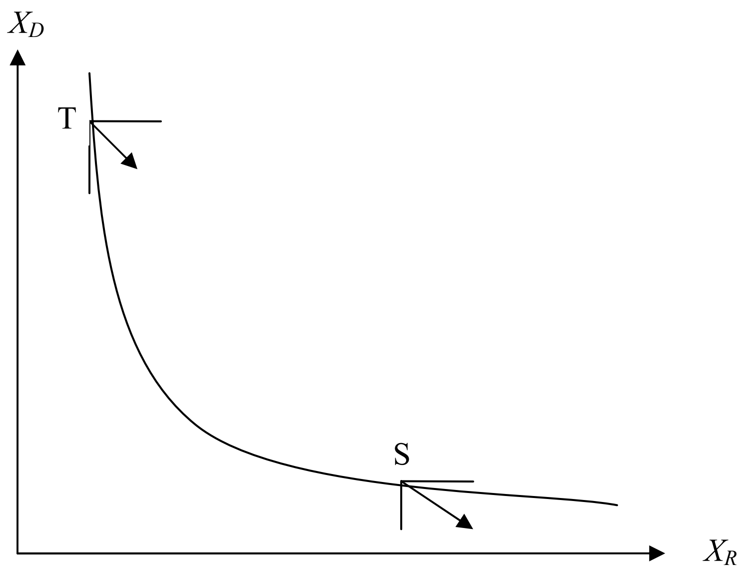


Figure 1: An Iso-welfare Curve